STORAGE OF HEAT BY A SEMI-INFINITE BODY EXPOSED TO A STEPWISE TEMPERATURE CYCLE

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The Laplace transform is used to determine the temperature field of a wall exposed to an asymmetric stepwise temperature cycle. A formula is obtained for the amount of heat stored in a semi-infinite slab. A graph is presented, together with numerical calculations based on the theory described.

Temperature fluctuations of the outside air play a decisive role in computing the heat storage capacity of buildings. We are in full agreement with the viewpoint stemming from discussion in the Journal of Engineering Physics that there is no need to consider the temperature wave in an enclosure caused by interruptions in heating. When we are dealing with inhabited buildings, this is undoubtedly correct, since the inside air temperature must be held constant. Certain technological processes may occur in a medium, however, which cause its temperature to vary in the stepwise fashion shown in Fig. 1.

A temperature regime of this kind must be maintained in chambers for fermenting sour milk, where the temperature must be alternated from 18 to 42° C every 2.5 hr. That is, the air temperature inside the chamber is increased or decreased by 12° in stepwise fashion, relative to a mean of 30°. This occasions considerable storage of heat, which should be taken into account in determining the dimensions of the thermal installations which must ensure the above temperature regime.

The temperatures are usually different in two adjoining chambers: when it is 42° in one chamber, it is 18° C in the other. Therefore the temperature field and the dividing wall are exposed to an asymmetric thermal effect. This field is described by the heat conduction equation

$$\frac{\partial t(x,\ \tau)}{\partial \tau} = a\,\frac{\partial^2 t(x,\ \tau)}{\partial x^2} \tag{1}$$

with the boundary conditions

$$t(x,\ 0) = 0, \quad t(0,\ \tau) = 0,$$
$$\lambda\,\frac{\partial t(\delta,\ \tau)}{\partial x} + \alpha\,[t(\delta,\ \tau) - t_f(\tau)] = 0. \tag{2}$$

The problem may be solved by the integral Laplace transform method. Since the transform of external temperatures $t_f(\tau)$, shown in Fig. 1, according to (1) and (2), has the form

$$T_f(s) = L[t_f(\tau)] = \frac{t_m}{s}\,\mathrm{th}\,\frac{Ts}{2}, \tag{3}$$

the general solution may be written in transforms as

$$T(x,\ s) = \frac{t_m\,\mathrm{sh}\,(Ts/2)\,\mathrm{sh}\sqrt{s/a}\,x}{s\,\mathrm{ch}\,(Ts/2)\,(\mathrm{sh}\sqrt{s/a}\,\delta + (\lambda/\alpha)\sqrt{s/a}\,\mathrm{ch}\sqrt{s/a}\,\delta)}. \tag{4}$$

For the zero pole, i.e., $s = 0$, the original temperature is equal to zero. The transient process is determined by the pole

$$s_n = -\mu_n^2\,a/\delta^2,$$

where μ are roots of the characteristic equation $\mathrm{tg}\,\mu = -\mu\,\mathrm{Bi}$. Since we are interested in the quasi-steady process, the required expression for the poles takes the form

$$s_n = \pm(2n+1)\,\frac{\pi}{T}\,i, \quad n = 0,\ 1,\ 2,\ \ldots \tag{5}$$

Applying the expansion theorem to (4), with allowance for (5), we find

$$\frac{t(x,\ \tau)}{t_m} = \sum_{n=0}^{\infty}\frac{2}{\pi(2n+1)}\left\{N_i \exp\left[(2n+1)\frac{\pi\tau}{T}\,i\right] + N_{-i}\exp\left[-(2n+1)\frac{\pi\tau}{T}\,i\right]\right\}, \tag{6}$$

where

$$N_i = \left(\mathrm{sh}\sqrt{i(2n+1)\frac{\pi}{aT}}\,x\right)\left[i\left(\mathrm{sh}\sqrt{i(2n+1)\frac{\pi\delta^2}{aT}} + \frac{\lambda}{\alpha\delta}\sqrt{i(2n+1)\frac{\pi\delta^2}{aT}}\,\mathrm{ch}\sqrt{i(2n+1)\frac{\pi\delta^2}{aT}}\right)\right]^{-1}, \tag{7}$$

and N_{-i} is described by (7), where the sign of i is replaced by the opposite.

If the origin of the coordinate system is transferred to the surface on the left, i.e., x is replaced by $X - \delta$ and we put $\delta \to \infty$, we obtain the solution for a semi-infinite slab. In this case (7) takes the form

$$N_i = \frac{\exp\left[-X\sqrt{i(2n+1)\frac{\pi}{aT}}\right]}{1 + \frac{\lambda}{\alpha}\sqrt{i(2n+1)\frac{\pi}{aT}}}\,i. \tag{8}$$

The heat stored by unit surface of the body is determined by

$$Q = \lambda\int_0^T \frac{\partial t(0,\ \tau)}{\partial x}\,d\tau. \tag{9}$$

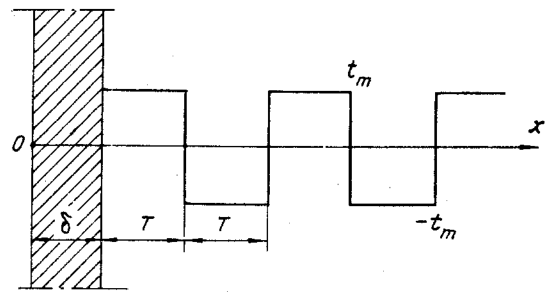

Fig. 1. Variation of the temperature of the medium.

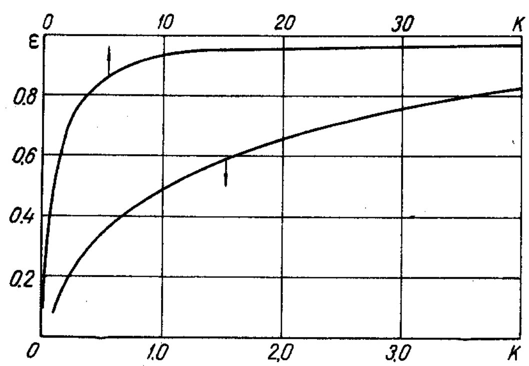

Fig. 2. Dependence of ε on $K=\sqrt{\pi\lambda c\gamma/\alpha^2 T}$.

Therefore, after substituting the solution for the semi-infinite slab and integrating, we find

$$Q = \alpha t_m T \varepsilon, \tag{10}$$

where

$$\varepsilon = \sum_{n=0}^{\infty} \left(\frac{2}{\pi (2n+1)} \right)^2 \frac{2k(\sqrt{2/(2n+1)} + 2k)}{(\sqrt{2/(2n+1)} + K)^2 + K^2}. \tag{11}$$

In Eq. (11) K takes the form

$$K = \sqrt{\pi \lambda c \gamma / \alpha^2 T}. \tag{12}$$

Analysis of the solutions obtained shows that the amplitude of the temperature oscillations, as calculated by the exact solution (6) is rapidly attenuated as we go into the wall. We may therefore use the simpler solutions represented by (10), which gives results that are close to the true values, although high. Moreover, from (10) we may calculate the heat stored in the floor and ceiling, although they do not experience asymmetric temperature effects.

Example of calculation. Take a chamber (3.6 × × 12.5 × 2 m) for fermenting sour milk, with a reinforced concrete floor and walls and ceiling of foam concrete. The process technology requires that the chamber temperature alternates between 18° and 42° C every 2.5 hr. It is required to determine the total amount of heat stored by the refrigerating chamber.

The thermophysical parameters have the following values for reinforced concrete: $\lambda = 1.55$ W/m · °C, c = 850 J/kg · °C, $\gamma = 2200$ kg/m^3. The heat transfer coefficient α between the floor and the air is taken to be 12 W/m^2 · °C; T = 9000 sec. Substituting these data in (12), we find K = 2.65. With this value of K we find from Fig. 2 that $\varepsilon = 0.72$. Therefore the heat stored by the floor is Q = 12 · 12 · · 9000 · 0.72 = 930 000 J/m^2 = 930 kj/m^2.

The thermophysical constants for foam concrete are: $\lambda = 0.17$ W/m · °C, c = 850 J/kg · °C, $\gamma = 300$ kg/m^3. The heat transfer coefficient α between the air and the walls is taken to be 24 W/m^2 · °C. From these data we find K = 0.162 and $\varepsilon = 0.15$. The amount of heat stored by the walls will be 390 kJ/m^2. Taking the heat transfer coefficient between the air and the ceiling to be $\alpha = 12$ W/m^2 · °C, we find K = 0.324, $\varepsilon = 0.26$, Q = 385 kJ/m^2.

Therefore the total amount of heat stored by the refrigerating chamber is Q = 82 000 kJ.

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